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Hypercharge and baryon minus lepton number in E_6

Junpei Harada

Department of Physics, Hiroshima University
Higashi-Hiroshima 739-8526, Japan
E-mail: harada@theo.phys.sci.hiroshima-u.ac.jp

Abstract: We study assignments of the hypercharge and baryon minus lepton number for particles in the E_6 grand unification model. It is shown that there are three assignments of hypercharge and three assignments of baryon minus lepton number which are consistent with the Standard Model. Their explicit expressions and detailed properties are given. In particular, we show that the $\mathrm{U}(1)_{B-L}$ symmetry in E_6 cannot be orthogonal to the $\mathrm{SU}(3)_R$ symmetry. Based on these investigations, we propose an alternative SU(5) grand unification model.

Keywords: Beyond Standard Model, GUT.

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Contents

1. Introduction

The supersymmetric (SUSY) grand unified theory (GUT) is one of the most attractive scenarios beyond the Standard Model (SM). Among all the possible SUSY GUT models, the minimal SUSY SU(5) model has been most extensively studied in the past. However, recent experimental progress suggests that it is no longer the primary candidate and some modifications are obviously required for several reasons. The first reason is that the lower limit on the proton lifetime measured by the Super-Kamiokande [1] already excludes the minimal SUSY SU(5) model [2].[1] The second reason is that small neutrino masses suggested by recent neutrino oscillation experiments [5] can be naturally explained by the seesaw mechanism [6], which requires right-handed neutrinos in addition to the minimal SM particles.

Although an extension to some non-minimal SUSY SU(5) GUT models is one possible solution, it is worth investigating GUT models with alternative gauge groups. Extending the gauge group beyond SU(5) is interesting since the theory includes the right-handed neutrinos and we have several ways of embedding the SM gauge group into the extended gauge group. For example, in the SO(10) GUT there are two different assignments of hypercharge, which are related with each other by the $SU(2)_R$ symmetry. This fact leads to two different GUT models in the context of the SU(5) group; the Georgi-Glashow model [7] or the flipped model [8, 9]. Recent studies show that the SUSY flipped SU(5) model overcomes the difficulties in the minimal SUSY SU(5) GUT [10].

The E_6 is an unique exceptional Lie group that has complex representations and gives an observed chiral structure at low-energies. The E_6 GUT [11] has been known to give, for example, the small Cabbibo-Kobayashi-Maskawa (CKM) mixing angles and bi-large neutrino mixings [12, 13]. Since E_6 is larger than SO(10), there may be other different assignments of hypercharge and baryon minus lepton number ($B - L$) in the E_6 grand unification. These possibilities lead to new alternative GUT models.

[1] There is still controversy in this statement. This issue is discussed in detail in refs. [3, 4].

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In this paper we thoroughly investigate assignments of the hypercharge and $B-L$ in the E_6 unification. We show that in the E_6 GUT there are *three* assignments of hypercharge [14] and *three* assignments of $B-L$ that reproduce the SM. Their properties are very important for model building. We point out that these three assignments are related with each other by the SU(2) subgroup of $\mathrm{SU}(3)_R$, which is the subgroup of E_6. We also show that each assignment is orthogonal to the SU(2) subgroup of $\mathrm{SU}(3)_R$. In particular, we emphasize that the $\mathrm{U}(1)_{B-L}$ symmetry in E_6 can not be orthogonal to the $\mathrm{SU}(3)_R$ symmetry. This fact strongly restricts the E_6 grand unified theories with the gauged $\mathrm{U}(1)_{B-L}$ symmetry. In those of $9(3\times 3)$ pairs of charge assignments, we show that 6 pairs are consistent with the SM. These observation indicate that E_6 has much potential for constructing alternative GUT models compared to the SO(10) GUT.

We also propose a E_6-inspired alternative SU(5) GUT model, "E-twisting flipped model". The gauge group of this model is $\mathrm{SU}(5)\times\mathrm{U}(1)_V\times\mathrm{U}(1)_{V'}$ which is the subgroup of E_6. Assignments of hypercharge and $B-L$ are different from those of the Georgi-Glashow model or usual flipped model. In this model, since the hypercharge is not a subgroup of SU(5), the quasi-unification of strong, weak and hypercharge gauge couplings is predicted. While Dirac masses of quarks or leptons are naturally obtained through the renormalizable operators in the superpotential, the heavy Majorana masses of right-handed neutrinos are obtained via dimension five operators. Therefore, the conventional seesaw mechanism can work.

This paper is organized as follows. In section 2 we review SO(10) GUT since it will be helpful when discussing E_6. Especially, we emphasize that as far as $\mathrm{SO}(10)\supset\mathrm{U}(1)_{B-L}$ is imposed, only the **16** representation, not **10**, can be a candidate for the SM matters in both the Georgi-Glashow model and usual flipped model. In section 3 we investigate the hypercharge and $B-L$ assignments in the E_6 unification. We show that there are three assignments of hypercharge or $B-L$. Their explicit expressions and detailed properties are given. In section 4, as an example, we propose a E_6-inspired alternative SU(5) GUT model. section 5 is devoted to the conclusion.

2. Particle assignment in SO(10)

A crucial difference between the SO(10) GUT and the SU(5) GUT resides in the fact that the way of embedding the SM gauge group into the unified gauge group is unique or not. In the SU(5) GUT it is unique, while in the SO(10) GUT there are two different assignments of hypercharge. Since different assignments lead to different predictions, this problem is important for low energy physics. In this section we study this issue in detail.

In the minimal SO(10) GUT, all matters in the SM and a right-handed neutrino belong to **16** for each family. Each spinor representation **16** is decomposed under a maximal subgroup $\mathrm{SO}(10)\supset\mathrm{SU}(5)\times\mathrm{U}(1)_V$ as

$$\mathbf{16}=\mathbf{5^*}_3+\mathbf{10}_{-1}+\mathbf{1}_{-5}\,, \tag{2.1}$$

and furthermore under $\mathrm{SU}(5) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Z$ as

$$\begin{aligned}
\mathbf{5}^* &= (\mathbf{3}^*, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2})_{-1/2} \,, \\
\mathbf{10} &= (\mathbf{3}, \mathbf{2})_{1/6} + (\mathbf{3}^*, \mathbf{1})_{-2/3} + (\mathbf{1}, \mathbf{1})_1 \,, \\
\mathbf{1} &= (\mathbf{1}, \mathbf{1})_0 \,.
\end{aligned} \tag{2.2}$$

Notice here that $\mathrm{U}(1)_Z$, which is the subgroup of SU(5), is not identical with the $\mathrm{U}(1)_Y$ hypercharge at this stage. Clearly, each **16** includes two pairs of $(\mathbf{3}^*, \mathbf{1})$ or $(\mathbf{1}, \mathbf{1})$. This fact can be understood by considering the decomposition of **16** under another maximal subgroup $\mathrm{SO}(10) \supset \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$,

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) \,, \tag{2.3}$$

and under $\mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$ as

$$\begin{aligned}
(\mathbf{4}, \mathbf{2}, \mathbf{1}) &= (\mathbf{3}, \mathbf{2}, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2}, \mathbf{1})_{-1} \,, \\
(\mathbf{4}^*, \mathbf{1}, \mathbf{2}) &= (\mathbf{3}^*, \mathbf{1}, \mathbf{2})_{-1/3} + (\mathbf{1}, \mathbf{1}, \mathbf{2})_1 \,.
\end{aligned} \tag{2.4}$$

This decomposition shows that two pairs of $(\mathbf{3}^*, \mathbf{1})$ or $(\mathbf{1}, \mathbf{1})$ are $\mathrm{SU}(2)_R$ doublets. One can also find that the quantum numbers of $\mathrm{U}(1)_Z$ and $\mathrm{U}(1)_V$ are represented by the one of $\mathrm{U}(1)_X$ and the third component I_{3R} of $\mathrm{SU}(2)_R$ as follows,

$$\begin{aligned}
V &= -4I_{3R} - 3X \,, \\
Z &= -I_{3R} + \frac{1}{2}X \,.
\end{aligned} \tag{2.5}$$

Note that the X-charge can also be identified with $B - L$,

$$B - L = X = -\frac{1}{5}(V - 4Z) \,. \tag{2.6}$$

Therefore, the $\mathrm{U}(1)_{B-L}$ symmetry in SO(10) is orthogonal to the $\mathrm{SU}(2)_R$ symmetry.

From above equations, one can find that SU(5) multiplets $\mathbf{5}^*$,**10** and **1** are formed as

$$\begin{aligned}
\mathbf{5}^*{}_3 &= (\mathbf{3}^*, \mathbf{1}, I_{3R} = -1/2)_{-1/3} + (\mathbf{1}, \mathbf{2}, \mathbf{1})_{-1} \,, \\
\mathbf{10}_{-1} &= (\mathbf{3}, \mathbf{2}, \mathbf{1})_{1/3} + (\mathbf{3}^*, \mathbf{1}, I_{3R} = 1/2)_{-1/3} + (\mathbf{1}, \mathbf{1}, I_{3R} = -1/2)_1 \,, \\
\mathbf{1}_{-5} &= (\mathbf{1}, \mathbf{1}, I_{3R} = 1/2)_1 \,.
\end{aligned} \tag{2.7}$$

These arguments indicate that the way of embedding SU(5) as $\mathrm{SO}(10) \supset \mathrm{SU}(5) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ is not unique [12]. As far as the $\mathrm{U}(1)_Y$ hypercharge is not considered, there is freedom of $\mathrm{SU}(2)_R$ rotation. It is worth mentioning here that there still remains $\mathrm{SU}(2)_R$ freedom of embedding SU(5) as $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_V \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_{B-L}$. This is based on the following relation; $\mathrm{U}(1)_{B-L} \perp \mathrm{SU}(2)_R$.

Let us consider the hypercharge assignments in the SO(10) GUT. The hypercharge $\mathrm{U}(1)_Y$ is not identical with $\mathrm{U}(1)_X$ and must be orthogonal to $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$. This fact shows that the $\mathrm{U}(1)_Y$ hypercharge is not orthogonal to $\mathrm{SU}(2)_R$. In other words, the assignment of the hypercharge eliminates the $\mathrm{SU}(2)_R$ freedom.

In order to give explicit expressions for the hypercharge, consider the following decomposition $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_V \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Z \times \mathrm{U}(1)_V$. Therefore, the hypercharge $\mathrm{U}(1)_Y$ must be a linear combination of $\mathrm{U}(1)_Z$ and $\mathrm{U}(1)_V$; $\mathrm{U}(1)_Y \subset \mathrm{U}(1)_Z \times \mathrm{U}(1)_V$. There are two assignments of hypercharge that reproduce the SM [8],

1. $\frac{Y}{2} = Z$; Georgi-Glashow model [7]
2. $\frac{Y}{2} = -\frac{1}{5}(Z+V)$; flipped model [8, 9].

These hypercharge assignments can also be expressed in terms of the third component of $\mathrm{SU}(2)_R$ and the quantum number of $\mathrm{U}(1)_X$ as

$$\frac{Y}{2} = Z = -I_{3R} + \frac{1}{2}X \,, \tag{2.8}$$

for the Georgi-Glashow model and

$$\frac{Y}{2} = -\frac{1}{5}(Z+V) = I_{3R} + \frac{1}{2}X \,, \tag{2.9}$$

for the flipped model.[2] These two assignments are different with each other in only the sign of the third component of $\mathrm{SU}(2)_R$. This means that the SU(5) group of flipped model is obtained from that of Georgi-Glashow model by the π rotation in $\mathrm{SU}(2)_R$ [12]. Namely the particle assignment of the flipped SU(5) model is given from that of the Georgi-Glashow SU(5) model by the interchange of $\mathrm{SU}(2)_R$ doublets (flipping),

$$u^c \leftrightarrow d^c \,, \qquad e^c \leftrightarrow \nu^c \,. \tag{2.10}$$

Although the way of embedding SU(5) as $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_V \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is not unique, there remains no $\mathrm{SU}(2)_R$ freedom anymore. Therefore, there are only two possibilities; SU(5) must be $\mathrm{SU}(5)_{GG}$ or $\mathrm{SU}(5)_{flipped}$.

Finally, we investigate **10** representations of SO(10). As mentioned above, in the minimal SO(10) GUT all matters in the SM and a right-handed neutrino belong to **16** for each family. However, since GUT models in which **10** includes the SM matters are possible, it is worth investigating **10** of SO(10). Each **10** representation is decomposed under a maximal subgroup $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_V$ as

$$\mathbf{10} = \mathbf{5}_2 + \mathbf{5^*}_{-2} \,, \tag{2.11}$$

and under $\mathrm{SU}(5) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Z$ as

$$\begin{aligned} \mathbf{5} &= (\mathbf{3},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{2})_{1/2} \,, \\ \mathbf{5}^* &= (\mathbf{3}^*,\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} \,. \end{aligned} \tag{2.12}$$

[2] These expressions are useful for comparing two different hypercharge assignments. The formula between hypercharge and $B-L$ can be always reduced to a conventional one $Y/2 = I_{3R} + (B-L)/2$ by redefining the third component of $\mathrm{SU}(2)_R$.

As before, consider the decomposition of **10** under another maximal subgroup $\mathrm{SO}(10) \supset \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ as

$$\mathbf{10} = (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2})\,, \tag{2.13}$$

and under $\mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$ as

$$\begin{aligned}(\mathbf{6},\mathbf{1},\mathbf{1}) &= (\mathbf{3},\mathbf{1},\mathbf{1})_{-2/3} + (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{2/3}\,,\\ (\mathbf{1},\mathbf{2},\mathbf{2}) &= (\mathbf{1},\mathbf{2},\mathbf{2})_0\,.\end{aligned} \tag{2.14}$$

From these equations, one can see that SU(5) multiplets **5** and $\mathbf{5}^*$ are formed as

$$\begin{aligned}\mathbf{5}_2 &= (\mathbf{3},\mathbf{1},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{2},I_{3R}=-1/2)_0\,,\\ \mathbf{5}^*{}_{-2} &= (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2},I_{3R}=1/2)_0\,.\end{aligned} \tag{2.15}$$

This indicates that if $\mathbf{5}^*$ of **10** is regarded as the SM matters, $\mathrm{U}(1)_X$ cannot be identified with $\mathrm{U}(1)_{B-L}$. We emphasize that as far as $\mathrm{SO}(10) \supset \mathrm{U}(1)_{B-L}$ is imposed, **10** of SO(10) must not be a candidate of the SM matters for both the Georgi-Glashow model and the flipped model. It is also noted here that even if $\mathrm{SO}(10) \supset \mathrm{U}(1)_{B-L}$ is not imposed, **10** can not be a candidate of the SM matters in the flipped model.[3]

3. Particle assignment in E_6

The E_6 GUT is one of the most attractive grand unified models and it is important for model building to investigate all the possible assignments of the hypercharge and $B-L$ in the E_6 unification. Although the E_6 GUT is more complicated than the SO(10) GUT since the rank of E_6 is larger, the argument is essentially the same as for the SO(10) GUT.

In the E_6 GUT, all matters in the SM (with a right-handed neutrino) and exotic matters belong to a fundamental representation **27** for each family. Each **27** representation is decomposed under a maximal subgroup $E_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)_{V'}$ as follows

$$\mathbf{27} = \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4\,. \tag{3.1}$$

These SO(10) multiplets **16**, **10** and **1** are decomposed under $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_V$ as

$$\begin{aligned}\mathbf{16} &= \mathbf{10}_{-1} + \mathbf{5}^*{}_3 + \mathbf{1}_{-5}\,,\\ \mathbf{10} &= \mathbf{5}_2 + \mathbf{5}^*{}_{-2}\,,\\ \mathbf{1} &= \mathbf{1}_0\,.\end{aligned} \tag{3.2}$$

The decomposition of SU(5) multiplets under $\mathrm{SU}(5) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Z$ is given in the previous section. One sees that each **27** includes three pairs of $(\mathbf{3}^*,\mathbf{1})$, $(\mathbf{1},\mathbf{2})$ or $(\mathbf{1},\mathbf{1})$, where the numbers in parentheses are the dimensions of $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ respectively.

[3] Here we mean "flipped model" as the model that is composed of SU(5) multiplets $\mathbf{5}^* = (u^c, e, \nu)$, $\mathbf{10} = (d^c, u, d, \nu^c)$ and $\mathbf{1} = e^c$.

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This can be understood by considering the decomposition of **27** under a maximal subgroup $E_6 \supset \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ as

$$\mathbf{27} = (\mathbf{3},\mathbf{3},\mathbf{1}) + (\mathbf{3}^*,\mathbf{1},\mathbf{3}^*) + (\mathbf{1},\mathbf{3}^*,\mathbf{3})\,, \tag{3.3}$$

and under $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_{Y_L}$ as

$$\begin{aligned}(\mathbf{3},\mathbf{3},\mathbf{1}) &= (\mathbf{3},\mathbf{2},\mathbf{1})_{1/2} + (\mathbf{3},\mathbf{1},\mathbf{1})_{-1}\,,\\ (\mathbf{3}^*,\mathbf{1},\mathbf{3}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{3}^*)_0\,,\\ (\mathbf{1},\mathbf{3}^*,\mathbf{3}) &= (\mathbf{1},\mathbf{2},\mathbf{3})_{-1/2} + (\mathbf{1},\mathbf{1},\mathbf{3})_1\,.\end{aligned} \tag{3.4}$$

These equations show that three pairs of $(\mathbf{3}^*,\mathbf{1})$, $(\mathbf{1},\mathbf{2})$ or $(\mathbf{1},\mathbf{1})$ are $\mathrm{SU}(3)_R$ triplets.

For later discussions, consider further decompositions under $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_{(R)} \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_{(R)}}$(the meaning of parentheses for $''R''$ will be clarified soon) as

$$\begin{aligned}(\mathbf{3},\mathbf{3},\mathbf{1}) &= (\mathbf{3},\mathbf{2},\mathbf{1})_{1/2,0} + (\mathbf{3},\mathbf{1},\mathbf{1})_{-1,0}\,,\\ (\mathbf{3}^*,\mathbf{1},\mathbf{3}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{2})_{0,-1/2} + (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{0,1}\,,\\ (\mathbf{1},\mathbf{3}^*,\mathbf{3}) &= (\mathbf{1},\mathbf{2},\mathbf{2})_{-1/2,1/2} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2,-1} + (\mathbf{1},\mathbf{1},\mathbf{2})_{1,1/2} + (\mathbf{1},\mathbf{1},\mathbf{1})_{1,-1}\,,\end{aligned} \tag{3.5}$$

where the numbers in parentheses are the dimensions of $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_{(R)}$ respectively, and subscripts represent the quantum numbers of $\mathrm{U}(1)_{Y_L}$ and $\mathrm{U}(1)_{Y_{(R)}}$ respectively. Notice here that there are three SU(2) subgroups of $\mathrm{SU}(3)_R$. We define three SU(2) subgroups of $\mathrm{SU}(3)_R$ as follows;[4] $(\mathbf{16},\mathbf{10},\mathbf{3}^*,\mathbf{1})$ and $(\mathbf{16},\mathbf{5}^*,\mathbf{3}^*,\mathbf{1})$ are $\mathrm{SU}(2)_R$ doublets, $(\mathbf{16},\mathbf{10},\mathbf{3}^*,\mathbf{1})$ and $(\mathbf{10},\mathbf{5}^*,\mathbf{3}^*,\mathbf{1})$ are $\mathrm{SU}(2)'_R$ doublets [15], $(\mathbf{16},\mathbf{5}^*,\mathbf{3}^*,\mathbf{1})$ and $(\mathbf{10},\mathbf{5}^*,\mathbf{3}^*,\mathbf{1})$ are $\mathrm{SU}(2)_E$ doublets [12], where the numbers in parentheses are the dimensions of $\mathrm{SO}(10), \mathrm{SU}(5), \mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$ respectively. The meaning of parentheses for $''R''$ is clear now, namely, $\mathrm{SU}(2)_{(R)}$ means $\mathrm{SU}(2)_R$, $\mathrm{SU}(2)'_R$ or $\mathrm{SU}(2)_E$.

The quantum numbers of $\mathrm{U}(1)_{V'}$, $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_Z$ can be represented by those of $\mathrm{U}(1)_{Y_L}$, $\mathrm{U}(1)_{Y_{(R)}}$ and the third component $I_{3(R)}$ of $\mathrm{SU}(2)_{(R)}$ as

$$\begin{aligned}V' &= 2Y_L - 2Y_R\,,\\ V &= -2Y_L - 4I_{3R} - 2Y_R\,,\\ Z &= \frac{1}{3}Y_L - I_{3R} + \frac{1}{3}Y_R\,,\end{aligned} \tag{3.6}$$

or

$$\begin{aligned}V' &= 2Y_L + 3I'_{3R} + Y'_R\,,\\ V &= -2Y_L + I'_{3R} + 3Y'_R\,,\\ Z &= \frac{1}{3}Y_L - I'_{3R} + \frac{1}{3}Y'_R\,,\end{aligned} \tag{3.7}$$

[4] Note that the definition of SU(2) subgroups of $\mathrm{SU}(3)_R$ is not unique. We point out that there is another useful definition and it will be discussed later.

or

$$
\begin{aligned}
V' &= 2Y_L + 3I_{3E} + Y_E\,, \\
V &= -2Y_L + 5I_{3E} - Y_E\,, \\
Z &= \frac{1}{3}Y_L - \frac{2}{3}Y_E\,.
\end{aligned} \tag{3.8}
$$

From these expressions, the SU(5) multiplets, which belong to a single **27** of E_6, are formed under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_R}$ as

$$
\begin{aligned}
(\mathbf{16},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I_{3R}=-1/2))_{0,-1/2} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2,-1}\,, \\
(\mathbf{16},\mathbf{10}) &= (\mathbf{3},\mathbf{2},\mathbf{1})_{1/2,0} + (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I_{3R}=1/2))_{0,-1/2} + (\mathbf{1},\mathbf{1},\mathbf{2}(I_{3R}=-1/2))_{1,1/2}\,, \\
(\mathbf{16},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{2}(I_{3R}=1/2))_{1,1/2}\,, \\
(\mathbf{10},\mathbf{5}) &= (\mathbf{3},\mathbf{1},\mathbf{1})_{-1,0} + (\mathbf{1},\mathbf{2},\mathbf{2}(I_{3R}=-1/2))_{-1/2,1/2}\,, \\
(\mathbf{10},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{0,1} + (\mathbf{1},\mathbf{2},\mathbf{2}(I_{3R}=1/2))_{-1/2,1/2}\,, \\
(\mathbf{1},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{1})_{1,-1}\,.
\end{aligned} \tag{3.9}
$$

and then under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)'_R \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)'_{Y_R}$ as

$$
\begin{aligned}
(\mathbf{16},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{0,1} + (\mathbf{1},\mathbf{2},\mathbf{2}(I'_{3R}=1/2))_{-1/2,1/2}\,, \\
(\mathbf{16},\mathbf{10}) &= (\mathbf{3},\mathbf{2},\mathbf{1})_{1/2,0} + (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I'_{3R}=1/2))_{0,-1/2} + (\mathbf{1},\mathbf{1},\mathbf{2}(I'_{3R}=-1/2))_{1,1/2}\,, \\
(\mathbf{16},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{1})_{1,-1}\,, \\
(\mathbf{10},\mathbf{5}) &= (\mathbf{3},\mathbf{1},\mathbf{1})_{-1,0} + (\mathbf{1},\mathbf{2},\mathbf{2}(I'_{3R}=-1/2))_{-1/2,1/2}\,, \\
(\mathbf{10},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I'_{3R}=-1/2))_{0,-1/2} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2,-1}\,, \\
(\mathbf{1},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{2}(I'_{3R}=1/2))_{1,1/2}\,,
\end{aligned} \tag{3.10}
$$

and furthermore under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_E \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_E}$ as

$$
\begin{aligned}
(\mathbf{16},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I_{3E}=1/2))_{0,-1/2} + (\mathbf{1},\mathbf{2},\mathbf{2}(I_{3E}=1/2))_{-1/2,1/2}\,, \\
(\mathbf{16},\mathbf{10}) &= (\mathbf{3},\mathbf{2},\mathbf{1})_{1/2,0} + (\mathbf{3}^*,\mathbf{1},\mathbf{1})_{0,1} + (\mathbf{1},\mathbf{1},\mathbf{1})_{1,-1}\,, \\
(\mathbf{16},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{2}(I_{3E}=-1/2))_{1,1/2}\,, \\
(\mathbf{10},\mathbf{5}) &= (\mathbf{3},\mathbf{1},\mathbf{1})_{-1,0} + (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2,-1}\,, \\
(\mathbf{10},\mathbf{5}^*) &= (\mathbf{3}^*,\mathbf{1},\mathbf{2}(I_{3E}=-1/2))_{0,-1/2} + (\mathbf{1},\mathbf{2},\mathbf{2}(I_{3E}=-1/2))_{-1/2,1/2}\,, \\
(\mathbf{1},\mathbf{1}) &= (\mathbf{1},\mathbf{1},\mathbf{2}(I_{3E}=1/2))_{1,1/2}\,.
\end{aligned} \tag{3.11}
$$

As far as $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L$ is concerned, there is $\mathrm{SU}(3)_R$ freedom. The relations among three SU(2) subgroups of $\mathrm{SU}(3)_R$ are given above.

Now we consider the hypercharge assignments in the E_6 unification. The hypercharge $\mathrm{U}(1)_Y$ must be a linear combination of $\mathrm{U}(1)_Z$, $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_{V'}$; $\mathrm{U}(1)_Y \subset \mathrm{U}(1)_Z \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{V'}$. There are *three* assignments of hypercharge that are consistent with the SM [14],

1. $\frac{Y}{2} = Z$,
2. $\frac{Y}{2} = -\frac{1}{5}(Z+V)$,
3. $\frac{Y}{2} = -\frac{1}{20}(4Z - V - 5V')$.

The $\mathrm{U}(1)_Y$ hypercharge for the first assignment or the second one is a subgroup of SO(10), while the last $\mathrm{U}(1)_Y$ is a subgroup of E_6. Their properties are very important for model building. Therefore, we express three assignments of hypercharge in terms of the quantum numbers of $\mathrm{U}(1)_{Y_L}$, $\mathrm{U}(1)_{Y_{(R)}}$ and the third component of $\mathrm{SU}(2)_{(R)}$ as

$$
\begin{aligned}
\frac{Y}{2} &= Z\,,\\
&= \frac{1}{3}Y_L - I_{3R} + \frac{1}{3}Y_R\,,\\
&= \frac{1}{3}Y_L - I'_{3R} + \frac{1}{3}Y'_R\,,\\
&= \frac{1}{3}Y_L - \frac{2}{3}Y_E\,,
\end{aligned}
\tag{3.12}
$$

or

$$
\begin{aligned}
\frac{Y}{2} &= -\frac{1}{5}(Z+V)\,,\\
&= \frac{1}{3}Y_L + I_{3R} + \frac{1}{3}Y_R\,,\\
&= \frac{1}{3}Y_L - \frac{2}{3}Y'_R\,,\\
&= \frac{1}{3}Y_L - I_{3E} + \frac{1}{3}Y_E\,,
\end{aligned}
\tag{3.13}
$$

or

$$
\begin{aligned}
\frac{Y}{2} &= -\frac{1}{20}(4Z - V - 5V')\,,\\
&= \frac{1}{3}Y_L - \frac{2}{3}Y_R,\\
&= \frac{1}{3}Y_L + I'_{3R} + \frac{1}{3}Y'_R\,,\\
&= \frac{1}{3}Y_L + I_{3E} + \frac{1}{3}Y_E\,.
\end{aligned}
\tag{3.14}
$$

These expressions show the properties of each assignment and the relations among them. The first hypercharge assignment $\mathrm{U}(1)_Y$ is *not* orthogonal to $\mathrm{SU}(2)_R$ or $\mathrm{SU}(2)'_R$, but *is* $\mathrm{SU}(2)_E$; $\mathrm{U}(1)_Y \not\perp \mathrm{SU}(2)_R$ or $\mathrm{SU}(2)'_R$, but $\mathrm{U}(1)_Y \perp \mathrm{SU}(2)_E$. The second assignment is ; $\mathrm{U}(1)_Y \not\perp \mathrm{SU}(2)_R$ or $\mathrm{SU}(2)_E$, but $\mathrm{U}(1)_Y \perp \mathrm{SU}(2)'_R$. The third assignment is ; $\mathrm{U}(1)_Y \not\perp \mathrm{SU}(2)'_R$ or $\mathrm{SU}(2)_E$, but $\mathrm{U}(1)_Y \perp \mathrm{SU}(2)_R$. Thus, we conclude that in the E_6 GUT any assignments of hypercharge are orthogonal to the SU(2) subgroup of $\mathrm{SU}(3)_R$. One can also find the relations among three hypercharge assignments. Since differences of three hypercharge assignments are only in the sign of the third component of $\mathrm{SU}(2)_{(R)}$, three assignments are related with each other by the π rotation of $\mathrm{SU}(2)_{(R)}$. Notice here that although the hypercharge assignment eliminates $\mathrm{SU}(3)_R$ freedom, $\mathrm{SU}(2)_{(R)}$ freedom still remain at this stage. This is different from the SO(10) GUT case.

Now we come to the quantum number $B-L$ in the E_6 unification. In this paper we concentrate on the case in which the $\mathrm{U}(1)_{B-L}$ symmetry is included in E_6 as a subgroup. First, consider the following subgroup $E_6 \supset \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \supset \mathrm{SU}(3)_C \times$

$SU(2)_L \times U(1)_{Y_L} \times SU(3)_R$. From eq. (3.4), $U(1)_{B-L}$ must not be identical with $U(1)_{Y_L}$. On the other hand, $U(1)_{B-L}$ must be orthogonal to $SU(3)_C$ and $SU(2)_L$. From these facts, we conclude that the $U(1)_{B-L}$ symmetry in E_6 must not be orthogonal to the $SU(3)_R$ symmetry;

$$U(1)_{B-L} \not\perp SU(3)_R \,. \tag{3.15}$$

This relation is valid as far as $E_6 \supset U(1)_{B-L}$ is imposed. Therefore, the remaining $SU(2)_{(R)}$ freedom disappears. This strongly restricts E_6 grand unified models with the gauged $U(1)_{B-L}$ symmetry.

If $E_6 \supset U(1)_{B-L}$ is imposed, $U(1)_{B-L}$ must be a linear combination of $U(1)_Z$, $U(1)_V$ and $U(1)_{V'}$; $U(1)_{B-L} \subset U(1)_Z \times U(1)_V \times U(1)_{V'}$. There are *three* assignments of $B-L$ that reproduce the SM;

1. $B-L = -\frac{1}{5}(V-4Z)$,
2. $B-L = \frac{1}{20}(16Z+V+5V')$,
3. $B-L = -\frac{1}{20}(8Z+3V-5V')$.

The properties of these $B-L$ assignments are understood by expressing it in terms of the charges of $U(1)_{Y_L}$, $U(1)_{Y_{(R)}}$ and the third component of $SU(2)_{(R)}$ as

$$\begin{aligned} B-L &= -\frac{1}{5}(V-4Z)\,, \\ &= \frac{2}{3}Y_L + \frac{2}{3}Y_R\,, \\ &= \frac{2}{3}Y_L - I'_{3R} - \frac{1}{3}Y'_R\,, \\ &= \frac{2}{3}Y_L - I_{3E} - \frac{1}{3}Y_E\,, \end{aligned} \tag{3.16}$$

or

$$\begin{aligned} B-L &= \frac{1}{20}(16Z+V+5V')\,, \\ &= \frac{2}{3}Y_L - I_{3R} - \frac{1}{3}Y_R\,, \\ &= \frac{2}{3}Y_L + \frac{2}{3}Y'_R\,, \\ &= \frac{2}{3}Y_L + I_{3E} - \frac{1}{3}Y_E\,, \end{aligned} \tag{3.17}$$

or

$$\begin{aligned} B-L &= -\frac{1}{20}(8Z+3V-5V')\,, \\ &= \frac{2}{3}Y_L + I_{3R} - \frac{1}{3}Y_R\,, \\ &= \frac{2}{3}Y_L + I'_{3R} - \frac{1}{3}Y'_R\,, \\ &= \frac{2}{3}Y_L + \frac{2}{3}Y_E\,. \end{aligned} \tag{3.18}$$

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fields	$SU(3)_C$	I_{3L}	Y	$B-L$	Q_{em}
u	$\mathbf{3}$	$1/2$	$1/3$	$1/3$	$2/3$
d	$\mathbf{3}$	$-1/2$	$1/3$	$1/3$	$-1/3$
u^c	$\mathbf{3}^*$	0	$-4/3$	$-1/3$	$-2/3$
d^c	$\mathbf{3}^*$	0	$2/3$	$-1/3$	$1/3$
ν	$\mathbf{1}$	$1/2$	-1	-1	0
e	$\mathbf{1}$	$-1/2$	-1	-1	-1
ν^c	$\mathbf{1}$	0	0	1	0
e^c	$\mathbf{1}$	0	2	1	1
D	$\mathbf{3}$	0	$-2/3$	$-2/3$	$-1/3$
E^c	$\mathbf{1}$	$1/2$	1	0	1
N^c	$\mathbf{1}$	$-1/2$	1	0	0
D^c	$\mathbf{3}^*$	0	$2/3$	$2/3$	$1/3$
N	$\mathbf{1}$	$1/2$	-1	0	0
E	$\mathbf{1}$	$-1/2$	-1	0	-1
S	$\mathbf{1}$	0	0	0	0

Table 1: The quantum numbers of left-handed fields that belong to **27** representation of E_6.

One can easily see that three $B-L$ assignments are orthogonal to the SU(2) subgroup of $SU(3)_R$ and relations among three assignments are similar to the case of the hypercharge, namely, they are related with each other by the π rotation of $SU(2)_{(R)}$.

Since there are three assignments of hypercharge and three assignments of $B-L$, $9(3\times 3)$ pairs of charge assignment exist. In those of 9 pairs, 6 pairs are consistent with the SM. Since the case in which the hypercharge and $B-L$ are orthogonal to the *same* SU(2) subgroup of $SU(3)_R$ is not consistent with the SM, 3 pairs of $U(1)_Y$ *and* $U(1)_{B-L} \perp SU(2)_{(R)}$ are removed.

We summarize the 6 pairs of charge assignment which are consistent with the SM;

1. $U(1)_Y \perp SU(2)_E$ and $U(1)_{B-L} \perp SU(2)_R$,
2. $U(1)_Y \perp SU(2)_E$ and $U(1)_{B-L} \perp SU(2)'_R$,
3. $U(1)_Y \perp SU(2)'_R$ and $U(1)_{B-L} \perp SU(2)_R$,
4. $U(1)_Y \perp SU(2)_R$ and $U(1)_{B-L} \perp SU(2)'_R$,
5. $U(1)_Y \perp SU(2)_R$ and $U(1)_{B-L} \perp SU(2)_E$,
6. $U(1)_Y \perp SU(2)'_R$ and $U(1)_{B-L} \perp SU(2)_E$.

We express particle assignments for each case in terms of SO(10). The quantum numbers of each field are given in the table 1.

The first assignment is the most familiar one;

$$\begin{aligned}\mathbf{16} &= (d^c + e + \nu) + (u^c + u + d + e^c) + \nu^c\,,\\ \mathbf{10} &= (D + E^c + N^c) + (D^c + E + N)\,,\\ \mathbf{1} &= S.\end{aligned} \tag{3.19}$$

The second assignment [16] is given from the first one by the π rotation in $\mathrm{SU}(2)_E$;

$$\begin{aligned} \mathbf{16} &= (D^c + E + N) + (u^c + u + d + e^c) + S\,, \\ \mathbf{10} &= (D + E^c + N^c) + (d^c + e + \nu)\,, \\ \mathbf{1} &= \nu^c\,. \end{aligned} \tag{3.20}$$

The third assignment is given from the first one by the π rotation in $\mathrm{SU}(2)_R$, namely the flipped model;

$$\begin{aligned} \mathbf{16} &= (u^c + e + \nu) + (d^c + u + d + \nu^c) + e^c\,, \\ \mathbf{10} &= (D + E + N) + (D^c + E^c + N^c)\,, \\ \mathbf{1} &= S\,. \end{aligned} \tag{3.21}$$

The fourth one is given from the third one by the π rotation in $\mathrm{SU}(2)_E$;

$$\begin{aligned} \mathbf{16} &= (D^c + E^c + N^c) + (d^c + u + d + \nu^c) + S\,, \\ \mathbf{10} &= (D + E + N) + (u^c + e + \nu)\,, \\ \mathbf{1} &= e^c\,. \end{aligned} \tag{3.22}$$

The fifth one is given from the first one by the π rotation in $\mathrm{SU}(2)'_R$;

$$\begin{aligned} \mathbf{16} &= (d^c + E^c + N^c) + (D^c + u + d + S) + \nu^c\,, \\ \mathbf{10} &= (D + e + \nu) + (u^c + E + N)\,, \\ \mathbf{1} &= e^c\,. \end{aligned} \tag{3.23}$$

The sixth one is given from the fifth one by the π rotation in $\mathrm{SU}(2)_E$;

$$\begin{aligned} \mathbf{16} &= (u^c + E + N) + (D^c + u + d + S) + e^c\,, \\ \mathbf{10} &= (D + e + \nu) + (d^c + E^c + N^c)\,, \\ \mathbf{1} &= \nu^c\,. \end{aligned} \tag{3.24}$$

These possibilities certainly indicate that E_6 has much potential for constructing new alternative GUT models.

Finally, we comment on the electromagnetic symmetry $\mathrm{U}(1)_{em}$. In the SM, the electromagnetic symmetry $\mathrm{U}(1)_{em}$ is, of course, a linear combination of $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$; $\mathrm{U}(1)_{em} \subset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. From eqs. (3.12)–(3.18), one can find that $Y/2 = -I_{3R} + (B-L)/2 = -I'_{3R} + (B-L)/2$ for the first assignment of the hypercharge, $Y/2 = I_{3R} + (B-L)/2 = -I'_{3E} + (B-L)/2$ for the second one, and $Y/2 = I'_{3R} + (B-L)/2 = I'_{3E} + (B-L)/2$ for the third one. These relations and eqs. (3.16)–(3.18) indicate $\mathrm{U}(1)_Y \subset \mathrm{SU}(2)_{(R)} \times \mathrm{U}(1)_{B-L}$ and $\mathrm{U}(1)_{B-L} \subset \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_{(R)}}$. This observation may indicate that the origin of the electromagnetic symmetry $\mathrm{U}(1)_{em}$ is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$; $\mathrm{U}(1)_{em} \subset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(2)_L \times \mathrm{SU}(2)_{(R)} \times \mathrm{U}(1)_{B-L} \subset \mathrm{SU}(2)_L \times \mathrm{SU}(2)_{(R)} \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_{(R)}} \subset \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$.[5]

[5]This relation becomes more simple form by redefining the SU(2) subgroups of $\mathrm{SU}(3)_R$; the SU(2) subgroup which is orthogonal to $\mathrm{U}(1)_Y$ is $\mathrm{SU}(2)_E$, the SU(2) subgroup which is orthogonal to $\mathrm{U}(1)_{B-L}$ is $\mathrm{SU}(2)_R$, and the remaining SU(2) subgroup is $\mathrm{SU}(2)'_R$. Using this definition the electromagnetic symmetry $\mathrm{U}(1)_{em}$ can be written as $\mathrm{U}(1)_{em} \subset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \subset \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{Y_L} \times \mathrm{U}(1)_{Y_R} \subset \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$.

4. Model

In this section, as a simple example, we propose a E_6-inspired SU(5) GUT model, "E-twisting flipped model". This model is based on the gauge group $\mathrm{SU}(5) \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{V'}$, which is a subgroup of E_6. We also assume $\mathcal{N} = 1$ supersymmetry. In this model, the hypercharge $\mathrm{U}(1)_Y$ is shared among SU(5), $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_{V'}$, and is explicitly given by

$$\frac{Y}{2} = -\frac{1}{20}(4Z - V - 5V'), \tag{4.1}$$

where notation is the same as in the previous section. This hypercharge $\mathrm{U}(1)_Y$ is orthogonal to the $\mathrm{SU}(2)_R$ symmetry. On the other hand, $B - L$ is given by

$$B - L = \frac{1}{20}(16Z + V + 5V'), \tag{4.2}$$

which is orthogonal to the $\mathrm{SU}(2)'_R$ symmetry.

All matters in the SM and a right-handed neutrino in a family belong to

$$\begin{aligned} F_{\mathbf{5}^*} &= \mathbf{5}^*{}_{-2,-2} = (u^c, e, \nu)\,, \\ T_{\mathbf{10}} &= \mathbf{10}_{-1,1} = (d^c, u, d, \nu^c)\,, \\ X_{\mathbf{1}} &= \mathbf{1}_{0,4} = e^c\,, \end{aligned} \tag{4.3}$$

where subscripts are the quantum numbers of $\mathrm{U}(1)_V$ and $\mathrm{U}(1)_{V'}$ respectively. At first glance, this particle assignments seems to be the same as those of usual flipped model. However, it is quite different. The $\mathrm{U}(1)_Y$ hypercharge and $B - L$ of our model are not subgroups of SO(10) such as usual flipped model, but of E_6. Since this model is given from the usual flipped model by the π rotation in $\mathrm{SU}(2)_E$ (E-twisting [12]), we call this model as "E-twisting flipped model". It is worth mentioning here that while SU(5) multiplets $\mathbf{5}^*$, $\mathbf{10}$ and $\mathbf{1}$ of usual flipped model can form a single $\mathbf{16}$ representation of SO(10), SU(5) multiplets in our model cannot. They must be $(\mathbf{10}, \mathbf{5}^*)$, $(\mathbf{16}, \mathbf{10})$ and $(\mathbf{1}, \mathbf{1})$, where the numbers in the parentheses are the dimensions of SO(10) and SU(5). Therefore, we introduce the following exotic matters to cancel anomaly; $\tilde{T}_{\mathbf{5}^*} \equiv \mathbf{5}^*{}_{3,1}$, $\tilde{F}_{\mathbf{5}} \equiv \mathbf{5}_{2,-2}$ and $\tilde{T}_{\mathbf{1}} \equiv \mathbf{1}_{-5,1}$. Although our model seems somehow complicated because of the existence of exotic matters, its structure is quite simple. Indeed, the SM matters and the exotic matters can form a single $\mathbf{27}$ representation of E_6. The exotic matters will decouple at low-energies. We assign the chiral superfields of the SM matters and exotic matters the odd Z_2 parity to avoid unwanted operators. This Z_2 parity can be regarded as an extended R-parity. Indeed, this extended R-parity is not identical with the conventional one $R = (-1)^{2S+3(B-L)}$, where B, L and S are the baryon, lepton and spin quantum numbers.

GUT symmetry breaking $\mathrm{SU}(5) \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{V'} \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is induced by Higgs multiplets $\Sigma_{\mathbf{10}} \equiv \mathbf{10}_{-1,1}$ and $\overline{\Sigma}_{\mathbf{10}^*} \equiv \mathbf{10}^*{}_{1,-1}$, where the electrically neutral components develop large vacuum expectation values (VEV), $\langle \nu^c_\Sigma \rangle = \langle \nu^c_{\overline{\Sigma}} \rangle \equiv M$. On the other hand, the Higgs multiplets which give Dirac masses to the SM matters are $\overline{\chi}_{\mathbf{5}^*} \equiv \mathbf{5}^*{}_{3,1}$ and $H_{\mathbf{5}} \equiv \mathbf{5}_{2,-2}$. Notice here that $\overline{\chi}_{\mathbf{5}^*}$ and $H_{\mathbf{5}}$ should be $(\mathbf{16}, \mathbf{5}^*)$ and $(\mathbf{10}, \mathbf{5})$ respectively. This is different from the case of Georgi-Glashow model or usual flipped model. We introduce other Higgs multiplets to cancel anomaly; $\phi_{\mathbf{1}} \equiv \mathbf{1}_{-5,1}$, $\overline{\phi}_{\mathbf{1}} \equiv \mathbf{1}_{5,-1}$, $\chi_{\mathbf{5}} \equiv \mathbf{5}_{-3,-1}$, $\overline{H}_{\mathbf{5}^*} \equiv \mathbf{5}^*{}_{-2,-2}$, $H'_{\mathbf{5}} \equiv \mathbf{5}_{2,2}$ and $\overline{H'}_{\mathbf{5}^*} \equiv \mathbf{5}^*{}_{-2,2}$. We assign Z_2 parity even to the Higgs chiral superfields.

The Z_2 parity and $\mathrm{SU}(5) \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{V'}$ invariant superpotential for the SM matters is

$$W = f_u T_{\mathbf{10}} F_{\mathbf{5}^*} \overline{\chi}_{\mathbf{5}^*} + f_d T_{\mathbf{10}} T_{\mathbf{10}} H_{\mathbf{5}} + f_l F_{\mathbf{5}^*} X_{\mathbf{1}} H_{\mathbf{5}} \, , \tag{4.4}$$

where we omit the indices of generations for simplicity, and f's are the Yukawa couplings. Thus, in this model, Dirac neutrino masses and up-type quark masses are the same each other; $m_u = m_{\nu_D}$, which is in contrast with the Georgi-Glashow model. On the other hand, although down-type quarks and charged leptons acquire Dirac masses from the same Higgs VEV, there is no relation among their Yukawa couplings. In other words, although there is no bottom-tau unification, there are also no wrong relations of $m_s = m_\mu$ and $m_d = m_e$. Right-handed Majorana masses are given by the following dimension five operator,

$$W = \frac{f_R}{M_{pl}} T_{\mathbf{10}} T_{\mathbf{10}} \overline{\Sigma}_{\mathbf{10}^*} \overline{\Sigma}_{\mathbf{10}^*} \, . \tag{4.5}$$

After GUT symmetry breaking, right-handed Majorana neutrinos acquire the following heavy masses,

$$M_R \sim \frac{f_R \langle \overline{\Sigma} \rangle^2}{M_{pl}} \, , \tag{4.6}$$

which leads to small neutrino masses through the conventional seesaw mechanism.

The superpotential for exotic matters is

$$W = y_D \tilde{T}_{\mathbf{5}^*} \tilde{F}_{\mathbf{5}} \phi_{\mathbf{1}} + \frac{y_S}{M_{pl}} \tilde{T}_{\mathbf{1}} \tilde{T}_{\mathbf{1}} \overline{\phi}_{\mathbf{1}} \overline{\phi}_{\mathbf{1}} \, . \tag{4.7}$$

After Higgs fields ϕ acquire VEVs, the exotic matters become super-heavy and decouple. Here we assume that VEVs of ϕ is slightly smaller than M.

Next we discuss the gauge coupling unification. First of all, notice that since the hypercharge assignment of our model is different from that of Georgi-Glashow model or flipped model, the gauge coupling flow is different from those of them. We normalize the charges and couplings in the following manner (where Z, V and V' are defined in the previous section), $\tilde{Z} \equiv \sqrt{3/5} Z$. The corresponding coupling is $g_{\tilde{Z}} = \sqrt{5/3} g_Z$. This coupling satisfies $g_{\tilde{Z}} = g_5$ at the SU(5) unification scale M_5. $\tilde{V} \equiv 1/2\sqrt{10} V$ and $g_{\tilde{V}} = 2\sqrt{10} g_V$. Note here that $tr_r \tilde{V}^2 = tr_r \tilde{Z}^2$ where r is any SO(10) representation. Therefore, we expect $g_{\tilde{V}} = g_5 = g_{10}$ at the SO(10) unification scale M_{10}. $\tilde{V'} \equiv 1/2\sqrt{6} V'$ and $g_{\tilde{V'}} = 2\sqrt{6} g_{V'}$. Note here that $tr_r \tilde{V'}^2 = tr_r \tilde{V}^2 = tr_r \tilde{Z}^2$ where r is any E_6 representation. We also expect $g_{\tilde{V'}} = g_{10} = g_6$ at the E_6 unification scale M_6. We can find the following relation at the SU(5) unification scale,

$$\frac{3}{5} \frac{1}{\alpha_Y} = \frac{1}{25} \frac{1}{\alpha_5} + \frac{3}{50} \frac{1}{\alpha_{\tilde{V}}} + \frac{9}{10} \frac{1}{\alpha_{\tilde{V'}}} \, , \tag{4.8}$$

where α_i is defined by $\alpha_i \equiv g_i^2/4\pi$ for any i. Therefore, if $24/25 \alpha_5^{-1} \sim 3/50 \alpha_{\tilde{V}}^{-1} + 9/10 \alpha_{\tilde{V'}}^{-1}$ is satisfied at the SU(5) unification scale, the quasi-unification of strong, weak and hypercharge gauge couplings will realize. Since it seems natural that the unification scale of SO(10) and E_6 are not much higher than that of SU(5), this situation should be possible. We comment on the ratio $K \equiv \alpha_1/\alpha_5$ at the scale M_5, where α_1 is defined by $\alpha_1 \equiv 5\alpha_Y/3$. The natural condition that $M_5 < M_{10} < M_6$ implies $\alpha_{\tilde{V}}(M_5) < \alpha_5(M_5)$ and $\alpha_{\tilde{V}'}(M_{10}) < \alpha_5(M_{10})$. From these relations and the infrared-free behavior of $\mathrm{U}(1)_{V'}$, we find the following inequality $\alpha_{\tilde{V}}, \alpha_{\tilde{V}'} < \alpha_5$ at the scale M_5. From both this relation and eq. (4.8), we conclude that $K < 1$. This is certainly a required relation that MSSM predicts from the precise experimental values at M_Z scale.

Although the detailed quantitative discussion and other phenomenological aspects of this model are interesting and should be pursued, that is beyond the scope of this paper.

Final comment is on the Higgs superpotential. We need, of course, a Higgs superpotential to define the complete model. We can easily write down the most general superpotential which is allowed by the gauge symmetry. However, there are some unwanted operators in the most general superpotential in view of F-flatness conditions and/or doublet-triplet splitting problem. In order to avoid unwanted operators, it is necessary to impose some additional symmetry beyond $\mathrm{SU}(5) \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{V'}$. Although there are some possibilities as an additional symmetry, we do not discuss here since they are model dependent. These possibilities will be investigated elsewhere.

5. Conclusion

In this paper we investigated assignments of the hypercharge and baryon minus lepton number for particles in the E_6 grand unification model. First we reviewed the SO(10) GUT and pointed out that as far as $\mathrm{SO}(10) \supset \mathrm{U}(1)_{B-L}$ is imposed, only the **16** representation, not **10**, can be a candidate for the SM matters. Next we studied the E_6 GUT and it was shown that there are three assignments of hypercharge and three assignments of $B - L$ which are consistent with the SM. Their explicit expressions and detailed properties were given. We showed that three assignments of hypercharge or $B - L$ are related with each other by the SU(2) subgroup of $\mathrm{SU}(3)_R$, which is the subgroup of E_6. We also pointed out that any charge assignments are orthogonal to the SU(2) subgroup of $\mathrm{SU}(3)_R$. In particular, we emphasized that the $\mathrm{U}(1)_{B-L}$ symmetry in E_6 can not be orthogonal to the $\mathrm{SU}(3)_R$ symmetry. This fact strongly restricts E_6 grand unified models with the gauged $\mathrm{U}(1)_{B-L}$ symmetry. In those of 9 pairs of charge assignments, we showed that 6 pairs are consistent with the SM. These observation indicate that E_6 has much potential for constructing alternative GUT models.

We also proposed a E_6-inspired SU(5) GUT model, "E-twisting flipped model". The charge assignments of hypercharge and $B - L$ in this model are different from those of Georgi-Glashow model or usual flipped model. Since the hypercharge is not a subgroup of SU(5), the quasi-unification of strong, weak and hypercharge gauge coupling is predicted. There are also no mass relations between down-type quarks and charged leptons.

Finally we would like to emphasize that it is worth investigating the alternative GUT models based on E_6 to overcome the difficulties of the minimal SUSY SU(5) model.

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